Total No. of Questions—**8**] [Total No. of Printed Pages—**4**

Seat No.	

[5667]-101

F.E. EXAMINATION, 2019

ENGINEERING MATHEMATICS—II

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

N.B. **:—** (*i*) Attempt Q. No. **1** or Q. No. **2**, Q. No. **3** or Q. No. **4**, Q. No. **5** or Q. No. **6**, Q. No. **7** or Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve the following differential equations :

(*i*) $(1+\log xy)\,dx+\left(1+\frac{x}{y}\right)dy=0$ [4]

(*ii*) $x^4\cdot\frac{dy}{dx}+x^3y-\sec(xy)=0$ [4]

(*b*) A long hollow pipe has an inner diameter of 10 cm and outer diameter of 20 cm. The inner surface is kept at 200°C and the outer surface at 50°C. The thermal conductivity is 0.12. Find the temperature at a distance 7.5 cm from the centre of the pipe. [4]

Or

2. (*a*) Solve $3y^2\frac{dy}{dx}+2xy^3=4xe^{-x^2}$ [4]

P.T.O.

(*b*) (*i*) A body at temperature 100°C is placed in a room whose temperature is 20°C and cools to 60°C in 5 minutes. Find its temperature after a further interval of 3 minutes. [4]

(*ii*) A resistance of 100 ohms, an inductance of 0.5 henry are connected in a series with a battery of 20 volts. Find the current in the circuit if $i = 0$ at $t = 0$. [4]

3. (*a*) Obtain the Fourier series expansion of $f(x) = x^3$, $-\pi < x < \pi$. [5]

(*b*) Evaluate $\int_0^\infty \sqrt{y}\, e^{-\sqrt{y}}\, dy$. [3]

(*c*) Trace the following curve (any *one*) : [4]

(*i*) $y^2(2a - x) = x^3$.

(*ii*) $r = a \cos 3\theta$

Or

4. (*a*) If $I_n = \int_{\pi/4}^{\pi/2} \cot^n \theta\, d\theta$, prove that :

$$I_n = \frac{1}{n-1} - I_{n-2}$$ [4]

(*b*) Prove that $\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} dx = \log\left(\frac{b}{a}\right)$ [4]

(*c*) Find the length of upper arc of one loop of lamniscate $r^2 = a^2 \cos 2\theta$ [4]

5. (*a*) Find the centre and radius of the circle of intersection of the sphere $x^2 + y^2 + z^2 - 2y - 4z - 11 = 0$ by the plane $x + 2y + 2z = 15$. [5]

(*b*) Find the equation of right circular cone which has its vertex at the point (0, 0, 10) and whose intersection with the plane XOY is a circle of diameter 10. [4]

(*c*) Find the equation of right circular cylinder whose axis is $x = 2y = -z$ and radius is 4. [4]

Or

6. (*a*) Find the equation of sphere through the circle $x^2 + y^2 + z^2 = 1$, $2x + 3y + 4z = 5$ and which intersects the sphere $x^2 + y^2 + z^2 + 3(x - y + z) - 56 = 0$ orthogonally. [5]

(*b*) Find the equation of right circular cone which passes through the point (2, –2, 1) with vartex at the (0, 0, 0) and axis parallel to the line $\frac{x-2}{5} = \frac{y-1}{1} = \frac{z+2}{1}$. [4]

(*c*) Find the equation of right circular cylinder of radius 2 whose axis passes through (1,2,3) and has direction cosines proportional to 2, –3, 6. [4]

7. Attempt any *two* of the following :

(*a*) Evaluate by changing the order of integration :

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{\cos^{-1} x \; dx\, dy}{\sqrt{(1-x^2-y^2)(1-x^2)}}$$ [7]

(*b*) Find the volume cut-off from the paraboloid $x^2 + \frac{y^2}{4} + z = 1$ by the plane $z = 0$. [6]

(*c*) Find the moment of inertia of one loop of Lemniscate $r^2 = a^2 \cos 2\theta$ about initial line. [6]

Or

8. Attempt any *two* of the following :

(*a*) Find the area inside the circle $r = a \sin \theta$ and outside the cardioid $r = a (1 - \cos \theta)$. [7]

(*b*) Evaluate :

$$\iiint_V \frac{dx\, dy\, dz}{\sqrt{1-x^2-y^2-z^2}} \quad [6]$$

taken throughout the volume of the sphere $x^2 + y^2 + z^2 = 1$ in the positive octant.

(*c*) Find the centre of gravity of an area of the cardioid $r = a (1 + \cos \theta)$ [6]