Total No. of Questions : 9]

SEAT No. :

PE-895

[Total No. of Pages : 4

[6581]-1901
F.E.
ENGINEERING MATHEMATICS - I
(2019 Pattern) (Semester - I) (107001)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Q. 1 is compulsory.*
2) *Attempt Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q,7, Q.8 or Q.9.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Figure to the right indicates full marks.*
5) *Use of electronic pocket calculator is allowed.*
6) *Assume suitable data, if necessary.*

Q1) Write the correct option for the following multiple choice questions. **[10]**

a) If z is a homogeneous function of x and y and if $z=f(u)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is **[1]**

i) $\frac{f(u)}{f'(u)}$ ii) $n\frac{f(u)}{f'(u)}$

iii) $\frac{f'(u)}{f(u)}$ iv) $n\frac{f'(u)}{f(u)}$

b) The product of eigen values of a matrix A is equal to **[1]**

i) Trace of A ii) Determinant of A

iii) Rank of A iv) None of the

c) The eigen values of $A=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$ are **[2]**

i) 0,0,0 ii) 0,0,1

iii) 0,0,3 iv) 1,1,1

d) The rank of the matrix $A=\begin{bmatrix}1 & 1 & 1\\ 1 & -1 & 0\\ 1 & 1 & 1\end{bmatrix}$ is **[2]**

i) 2 ii) 0

iii) 1 iv) 3

e) If $u = x^2 + y^2$, $v = 2xy$ then $\dfrac{\partial(u,v)}{\partial(x,y)}$ is **[2]**

i) $x^2 - y^2$ ii) $2(x^2-y^2)$

iii) $4(x^2+y^2)$ iv) $4(x^2-y^2)$

f) If $x = r\cos\theta$ and $y = r\sin\theta$ then $\left(\dfrac{\partial y}{\partial r}\right)_x$ is **[2]**

i) $\dfrac{r^2}{y}$ ii) $\dfrac{r}{y^2}$

iii) $\dfrac{r^2}{y^2}$ iv) $\dfrac{r}{y}$

Q2) a) If $v=\dfrac{c}{\sqrt{t}}e^{\left(-x^2/4a^2t\right)}$ then show that $\dfrac{\partial v}{\partial t}=a^2\dfrac{\partial^2 v}{\partial x^2}$. **[5]**

b) If $u=f\left(\dfrac{y}{x}\right)+\sqrt{x^2+y^2}$ prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\left(x^2+y^2\right)^{1/2}$. **[5]**

c) If $u=f(x^2-y^2, y^2-z^2, z^2-x^2)$ then find the value of $\dfrac{1}{x}f_x+\dfrac{1}{y}f_y+\dfrac{1}{z}f_z$. **[5]**

OR

Q3) a) If $x^2 = au + bv$, $y^2 = au - bv$ prove that $\left(\dfrac{\partial u}{\partial x}\right)_y\cdot\left(\dfrac{\partial x}{\partial u}\right)_v=\left(\dfrac{\partial v}{\partial y}\right)_x\cdot\left(\dfrac{\partial y}{\partial v}\right)_u$ **[5]**

b) If $u=\sec^{-1}\left(\dfrac{x^3+y^3}{x+y}\right)$ prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=2\cot u$. **[5]**

c) If $z=f(x,y)$ and $x=e^u+e^{-v}$, $y=e^{-u}-e^v$ show that $\dfrac{\partial z}{\partial u}-\dfrac{\partial z}{\partial v}=x\dfrac{\partial z}{\partial x}-y\dfrac{\partial z}{\partial y}$. **[5]**

Q4) a) If $x = u(1 - v)$, $y = uv$, prove that $JJ' = 1$. **[5]**

b) Prove that the functions $u = y + z$, $v = x + 2z^2$, $w = x - 4yz - 2y^2$ are functionally dependent and find the relation between them. **[5]**

c) Find the maximum and minimum values of $x^2 + y^2 + 8y + 15$. **[5]**

OR

Q5) a) If $u^3+v^3 = x + y$, $u^2 + v^2 = x^3+y^3$ Find $\frac{\partial(x,y)}{\partial(u,v)}$. **[5]**

b) A power dissipated in a resistor is given by $P = \frac{E^2}{R}$. Using calculus, find the approximate percentage error in P when E is increased by 3% and R is increased by 2%. **[5]**

c) Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third may be maximum. **[5]**

OR

Q6) a) Determine values of k for which the equations **[5]**
$2x - 3y + 5z = k$, $3x + y - z = 2$, $x + 4y - 6z = 1$ are consistent.

b) Examine following set of vectors for linearly dependance. **[5]**
$X_1 = (1,2,4)$, $X_2 = (2,-1,3)$, $X_3 = (0,1,2)$, $X_4 = (-3,7,2)$.

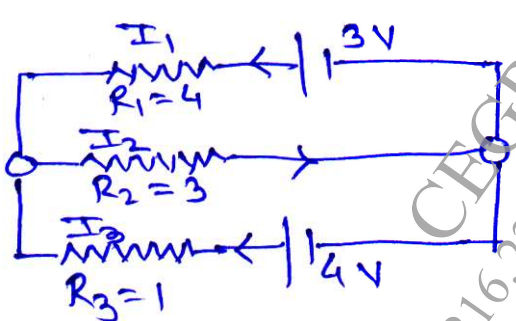
c) Show that $A = \begin{bmatrix} 1/\sqrt{3} & 0 & 2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{2} & -1/\sqrt{6} \\ -1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix}$ is an orthogonal matrix. **[5]**

OR

Q7) a) Solve the system of equations **[5]**
$x + y + 3z = 0$; $x - y + z = 0$; $-x + 2y = 0$; $x - y + 2z = 0$

b) Examine whether the set of vectors $X_1 \equiv (5,3,2)$, $X_2 \equiv (2,1,1)$, $X_3 \equiv (-3,1,6)$ are linearly independent. **[5]**

c) Find the current I_1, I_2, I_3 in the circuit shown in the figures : **[5]**



Q8) a) Find eigen values and eigen vectors for the given matrix $A = \begin{bmatrix} 3 & 5 \\ -2 & -4 \end{bmatrix}$. **[5]**

b) Verify Cayley Hamilton theorem for following matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. **[5]**

c) Find the modal matrix P which diagonalises the following matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$. **[5]**

OR

Q9) a) Find eigen values and eigen vector corresponding to largest eigen value of a following matrix $A = \begin{bmatrix} 15 & 0 & -15 \\ -3 & 6 & 9 \\ 5 & 0 & -5 \end{bmatrix}$ **[5]**

b) Verify Cayley Hamilton theorem for following matrix and hence find A^{-1} $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. **[5]**

c) Express the following quadratic form as "Sum of the squares form" by congruent transformation. Write down the corresponding linear transformation. **[5]**

$$Q(x) = 6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 + 4x_1x_3 - 2x_2x_3$$

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