

Total No. of Questions : 9]

SEAT No. :

P-3926

[Total No. of Pages : 5

[6001]-4001

F.E.

ENGINEERING MATHEMATICS - I
(2019 Pattern) (Semester - I) (107001)

Time : 2½ Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Question No. 1 is compulsory.
- 2) Solve Q. No. 2 or Q. No. 3, Q. No. 4 or Q. No. 5, Q. No. 6 or Q. No. 7, Q. No. 8 or Q. No. 9.
- 3) Neat diagrams must be drawn wherever necessary.
- 4) Figures to the right indicate full marks.
- 5) Electronic pocket calculator is allowed.
- 6) Assume suitable data, if necessary.

Q1) Write the correct option for the following multiple choice questions :

a) If $u = \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{x^2 + y^2}$, then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ is equal to [2]

- | | |
|--------|----------|
| i) 2u | ii) -2u |
| iii) 0 | iv) None |

b) If $u = x^y$ then $\frac{\partial u}{\partial y}$ is equal to [1]

- | | |
|-------------------|----------------|
| i) 0 | ii) yx^{y-1} |
| iii) $x^y \log x$ | iv) x^{y-1} |

c) If $x = uv$, $y = \frac{u}{v}$ then the value of $\frac{\partial(u,v)}{\partial(x,y)}$ is [2]

- | | |
|---------------------|---------------------|
| i) $\frac{-2u}{v}$ | ii) uv |
| iii) $\frac{v}{2u}$ | iv) $\frac{-v}{2u}$ |

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- d) A is orthogonal matrix then A^{-1} equal to [1]
 i) A ii) A^T
 iii) A^2 iv) 1
- e) For what value of K the homogeneous system $x + 2y - z = 0$,
 $3x + 8y - 3z = 0$; $2x + 4y + (k-3)z = 0$ has infinitely many solution. [2]
 i) $K = 0$ ii) $K = 1$
 iii) $K = 2$ iv) $K = 3$
- f) Using Cayley Hamilton theorem A^{-1} for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ is
 calculated from [2]
 i) $\frac{1}{5}(A - 4I)$ ii) $\frac{1}{5}(A - 4I)$
 iii) $\frac{2}{5}(A + 4I)$ iv) $\frac{1}{5}(4I - A)$

Q2) a) If $u = \ln(x^2 + y^2)$, show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$. [5]

b) If $e^{2u} = y^2 - x^2$, $\operatorname{cosec} v = \frac{y}{x}$ then find the value of [5]

$$\left(x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} \right) \cdot \left(x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y} \right)$$

c) If $u = f(x - y, y - z, z - x)$ then find the value of $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$. [5]

OR

Q3) a) If $u = ax + by$, $v = bx - ay$ find the value of $\left(\frac{\partial u}{\partial x} \right)_y \cdot \left(\frac{\partial v}{\partial y} \right)_x$. [5]

b) If $T = \sin\left(\frac{xy}{x^2 + y^2}\right) + \sqrt{x^2 + y^2}$, find the value of $x \frac{\partial T}{\partial x} + y \frac{\partial T}{\partial y}$. [5]

c) If $u = f(r, s)$ where $r = x^2 + y^2$, $s = x^2 - y^2$ then show that
 $y \frac{\partial u}{\partial x} + x \frac{\partial u}{\partial y} = 4xy \frac{\partial u}{\partial r}$. [5]

Q4) a) If $x = u + v$, $y = v^2 + w^2$, $z = u^3 + w^5$ then find $\frac{\partial u}{\partial x}$. [5]

b) In calculating resistance R of a circuit by using the formula :

$$R = \frac{V}{I}$$

errors of 3% and 1% are made in measuring Voltage V and current I respectively. Find the % error in the calculated resistance. [5]

c) Discuss the maxima and minima of : [5]

$$f(x, y) = x^2 + y^2 + xy + x - 4y + 5$$

OR

Q5) a) If $u + v^2 = x$, $v + w^2 = y$, $w + u^2 = z$ find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ [5]

b) Examine for functional dependence : [5]

$$u = y + z, v = x + 2z^2, w = x - 4yz - 2y^2$$

c) A space probe in the shape of the ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters the earth's atmosphere and its surface begins to heat. After one hour, the temperature at the point (x, y, z) on the surface of the probe is

$$T(x, y, z) = 8x^2 + 4yz - 16z + 600.$$

Find the hottest point on the surface of the probe, by using Lagrange's method. [5]

Q6) a) Examine for consistency and if consistent then solve it [5]

$$2x + 3y + 5z = 1 ; 3x + y - z = 2 ; x + 4y - 6z = 1$$

b) Examine whether the vectors [5]

$$X_1 = (1, 1, -1, 1); X_2 = (1, -1, 2, -1); X_3 = (3, 1, 0, 1)$$

are linearly independent or dependent. If dependent find relation between them.

c) If $A = \begin{bmatrix} 1/3 & 2/3 & a \\ 2/3 & 1/3 & b \\ 2/3 & -2/3 & c \end{bmatrix}$ is orthogonal [5]

Find a, b, c.

OR

Q7) a) Investigate for what values of k , the equations [5]

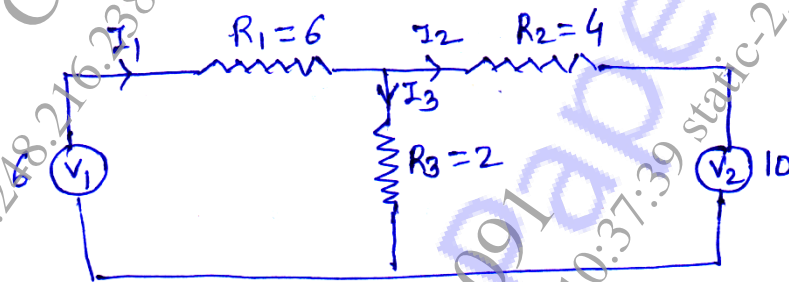
$x + y + z = 1$; $2x + y + 4z = k$; $4x + y + 10z = k^2$ have infinite number of solution? Hence find solution.

b) Examine whether the vectors. [5]

$X_1 = (2, 3, 4, -2)$; $X_2 = (-1, -2, -2, 1)$; $X_3 = (1, 1, 2, -1)$

are linearly independent or dependent. If dependent find relation between them.

c) Find the current I_1 ; I_2 ; I_3 in the circuit shown in the figure [5]



Q8) a) Find eigen values and eigen vectors of the following matrix [5]

$$A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

b) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and use it to find A^{-1} . [5]

c) Find the modal matrix p which transform the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ to the diagonal form. [5]

OR

Q9) a) Find eigen values and eigen vectors of the following matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. [5]

b) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ and use it to find A^{-1} [5]

c) Reduce the following quadratic form to the "sum of the squares form". [5]

$$Q(x) = 2x^2 + 9y^2 + 6z^2 + 8xy + 8yz + 6xz$$

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