Total No. of Questions : 9]

SEAT No. :

P6485

[Total No. of Pages : 4

[5868]-101

F.E. (Semester- I & II)

ENGINEERING MATHEMATICS - I

(2019 Pattern) (107001)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Q. 1 is compulsory.*
2) *Attempt Q2 or Q3, Q4 or Q5,Q6 or Q7, Q8 or Q9.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Figures to the right indicate full marks.*
5) *Use of electronic pocket calculator is allowed.*
6) *Assume suitable data, if necessary.*

Q1) Write the correct option for the following multiple choice questions.

a) If eigen value of a square matrix A is zero then. **[1]**

i) A is non-singular   ii) A is orthogonal

iii) A is singular   iv) None of these

b) If u = y^x then $\frac{\partial u}{\partial x}$ is equal to **[1]**

i) 0   ii) xy^{x-1}

iii) $y^x \log y$ iv) None of these

c) The orthogonal transformation x = py transforms the quadratic form $Q = x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2x_3$ to the canonical form $Q' = y_1^2 + 2y_2^2 + y_3^2$. The rank of quadratic from is **[2]**

i) 2   ii) 3

iii) 1   iv) 0

d) $u = \sec^{-1}\left[\frac{x^2 + y^2}{xy^2}\right]$. Find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ **[2]**

i) –tan u   ii) –cot u

iii) tan u   iv) cot u

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e) If u = $x^2 - y^2$ and v = $2xy$ then the value of $\frac{\partial(u,v)}{\partial(x,y)}$ is **[2]**

i) $4(x^2 + y^2)$
ii) $-4(x^2 + y^2)$
iii) $4(x^2 - y^2)$
iv) 0

f) A system of linear equations Ax = B, where B is a null (zero) matrix is **[2]**

i) Always consistent
ii) Consistent only if |A| = 0
iii) Consistent only if |A| $\neq$ 0
iv) In consistent if ρ (A) < No. of variables

Q2) a) If $z = \tan(y + ax) + (y - ax)^{3/2}$ find value of $\frac{\partial^2 z}{\partial x^2} - a^2 \frac{\partial^2 z}{\partial y^2}$. **[5]**

b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$ then prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \left(1 - 4\sin^2 u\right)\sin 2u$$ **[5]**

c) If $u = f\left(x^2 - y^2; y^2 - z^2; z^2 - x^2\right)$ find value of $\frac{1}{x}\frac{\partial u}{\partial x} + \frac{1}{y}\frac{\partial u}{\partial y} + \frac{1}{z}\frac{\partial u}{\partial z}$ **[5]**

OR

Q3) a) If $u = ax + by; v = bx - ay$ find value of $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v \left(\frac{\partial y}{\partial v}\right)_x \left(\frac{\partial v}{\partial y}\right)_u$ **[5]**

b) If $u = \sin^{-1}\left(\sqrt{x^2 + y^2}\right)$ then find value of $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$ **[5]**

c) If $u = f(r,s)$ where $r = x^2 + y^2$; S = $x^2 - y^2$ then show that

$$y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = 4xy\frac{\partial u}{\partial r}.$$ **[5]**

Q4) a) If $x = uv$ and $y = \frac{u+v}{u-v}$, find $\frac{\partial(u,v)}{x,y}$. **[5]**

b) Examine for functional dependence $u = \frac{x-y}{1+xy}, v = \tan^{-1}x - \tan^{-1}y$ and if depedent find the relation between them. **[5]**

c) Discuss maxima and minima of $f(x,y) = x^2 + y^2 + 6x + 12$ **[5]**

OR

Q5) a) Prove $JJ' = 1$ for $x = u \cos v$, $y = u \sin v$. **[5]**

b) In calculating the volume of a right circular cone, errors of 2% and 1% are made in measuring the height and radius of base respectively find the error in the calculated volume. **[5]**

c) Find maximum value of $u = x^2y^3z^4$ such that $2x + 3y + 4z = a$ by Langrange's method. **[5]**

Q6) a) Investigate for what values of μ & λ the equations $x+y+z = 6$, $x+2y+3z = 10$, $x+2y+\lambda z = \mu$ have i) No solution ii) Infinitely many solutions. **[5]**

b) Examine for linear dependence and independence the vectros (1,1,3), (1,2,4), (1,0,2). If dependent, find the relation between them. **[5]**

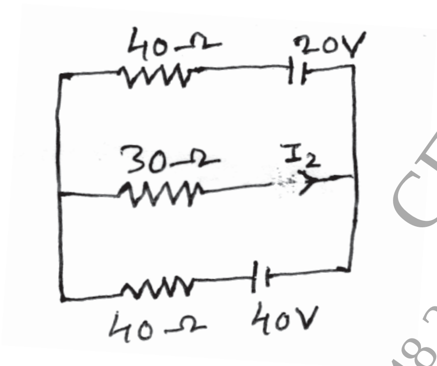
c) Verify whether matrix $A = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$ is orthogonal or not. **[5]**

OR

Q7) a) Solve the system of equations $x+y+2z = 0$, $x+2y+3z=0$, $x+3y+4z=0$. **[5]**

b) Examine following vectors for linear dependence and independence (1,–1,1), (2,1,1), (3,0,2). If dependent, find the relation between them. **[5]**

c) Determine the currents in the network given in the figure. **[5]**



Q8) a) Find the eigen values of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$. **[5]**

Find eigen vector corresponding to the highest eigen value.

b) Verify cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Hence find A^{-1} if it exists. **[5]**

c) Find the modal matrix p which diagonalises $A = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$. **[5]**

OR

Q9) a) Find the eigen values of $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$. **[5]**

Find eigen vector corresponding to the highest eigen value.

b) Verify cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ **[5]**

c) Reduce the quadratic form $Q = x_1^2 + 2x_2^2 + x_3^2 + 2x_2x_3 - 2x_3x_1 + 2x_1x_2$ to canonical form by congruent transformations. **[5]**

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