Seat No.

[5151]-101

F.E. EXAMINATION, 2017

ENGINEERING MATHEMATICS–I

(2015 PATTERN)

Time : Three Hours   **Maximum Marks : 50**

N.B. :— (*i*)   Attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6,** Q. No. **7** *or* Q. No. **8**.

(*ii*)   Neat diagrams must be drawn wherever necessary.

(*iii*)   Figures to the right indicate full marks.

(*iv*)   Use of electronic pocket calculator is allowed.

(*v*)   Assume suitable data, if necessary.

1.   (*a*)   Show that system of Linear equations is consistent. Find solution :   [4]

$$x + 2y + 3z = 6$$
$$2x + 3y = 11$$
$$4x + y - 5z = -3 \ .$$

(*b*)   Find eigen values and eigen vectors of the matrix :   [4]

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}.$$

(*c*)   Prove that :   [4]

$$(\cosh x - \sinh x)^n = \cosh nx - \sinh nx \ .$$

Or

2. (*a*) Are the following vectors are linearly dependent ? If so find relation : [4]

$$X_1 = (3, \ 2, \ 7),$$
$$X_2 = (2, \ 4, \ 1),$$
$$X_3 = (1, \ -2, \ 6).$$

(*b*) If α, β are roots of equation $x^2 - 2x + 4 = 0$, prove that : [4]

$$\alpha^n + \beta^n = 2^{n+1} \cos \frac{n\pi}{3}.$$

(*c*) If [4]

$$(a + ib)^p = m^{x+iy},$$

prove that :

$$\frac{y}{x} = \frac{2 \tan^{-1} \dfrac{b}{a}}{\log \ (a^2 + b^2)}.$$

3. (*a*) Test the convergence of the series (any *one*) : [4]

(*i*) $\displaystyle \sum \frac{x^n}{a + \sqrt{n}}$

(*ii*) $\dfrac{1}{\sqrt{5}} - \dfrac{1}{2\sqrt{6}} + \dfrac{1}{3\sqrt{7}} - \ \dots\dots$

(*b*) Show that : [4]

$$\log \left[\frac{1 + e^{2x}}{e^x} \right] = \log 2 + \frac{x^2}{2} - \frac{x^4}{12} + \frac{x^6}{45} - \dots\dots.$$

(*c*) Find the nth derivative of : [4]

$$y = e^x \cos x . \cos 2x .$$

4. (*a*) Solve any *one* : [4]

(*i*) $\lim\limits_{x \to 0} \left[\dfrac{\pi}{4x} - \dfrac{\pi}{2x(e^{\pi x} + 1)} \right]$

(*ii*) $\lim\limits_{x \to 0} \left(\dfrac{2^x + 5^x + 7^x}{3} \right)^{1/x}$.

(*b*) Using Taylor's theorem expand $x^3 - 2x^2 + 3x + 1$ in powers of $(x - 1)$. [4]

(*c*) If $y = e^{a \sin^{-1} x}$, prove that : [4]

$$(1 - x^2)y_{n+2} - (2n + 1)x \, y_{n+1} - (n^2 + a^2)y_n = 0.$$

5. Solve any *two* :

(*a*) Find the value of *n* for which $z = t^n e^{-r^2/4t}$ satisfies the partial differential equation : [6]

$$\frac{1}{r^2} \left[\frac{\partial}{\partial r} \left(r^2 \, \frac{\partial z}{\partial r} \right) \right] = \frac{\partial z}{\partial t}.$$

(*b*) If

$$T = \sin \left(\frac{xy}{x^2 + y^2} \right) + \sqrt{x^2 + y^2} + \frac{x^2 y}{x + y},$$

find the value of $x \dfrac{\partial T}{\partial x} + y \dfrac{\partial T}{\partial y}$. [7]

(c)   If $z = f(x, y)$, where

$$x = u \cos \alpha - v \sin \alpha,$$

$$y = u \sin \alpha - v \cos \alpha,$$

where α is constant, show that :   [6]

$$\left(\frac{\partial z}{\partial x} \right)^2 + \left(\frac{\partial z}{\partial y} \right)^2 = \left(\frac{\partial z}{\partial u} \right)^2 + \left(\frac{\partial z}{\partial v} \right)^2.$$

Or

6.   Solve any *two* :

(a)   If   [6]

$$x^2 = a\sqrt{u} + b\sqrt{v} \text{ and}$$

$$y^2 = a\sqrt{u} - b\sqrt{v}$$

where a and b are constants, prove that :

$$\left(\frac{\partial u}{\partial x} \right)_y \left(\frac{\partial x}{\partial u} \right)_v = \frac{1}{2} = \left(\frac{\partial v}{\partial y} \right)_x \left(\frac{\partial y}{\partial v} \right)_u.$$

(b)   If

$$u = \tan^{-1} \left(\frac{\sqrt{x^3 + y^3}}{\sqrt{x} + \sqrt{y}} \right),$$

then show that :   [7]

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -2 \sin^3 u \cos u.$$

(c)   If

$$u = x^2 - y^2, \quad v = 2xy \quad \text{and} \quad z = f(u, v),$$

then show that :   [6]

$$x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 2\sqrt{u^2 + v^2}\ \frac{\partial z}{\partial u}.$$

7.   (a)   If   [4]

$$u + v = x^2 + y^2, \quad u - v = x + 2y$$

Find $\dfrac{\partial u}{\partial x}$ treating y constant.

(b)   Examine for functional dependence :   [4]

$$u = \frac{x - y}{x + z}, \quad v = \frac{x + z}{y + z}.$$

(c)   Finsd stationary points of :   [5]

$$f(x, y) = x^3 y^2 (1 - x - y)$$

and find $f_{\max}$ where it exists.

Or

8.   (a)   If   [4]

$$x = v^2 + w^2, \quad y = w^2 + u^2, \quad z = u^2 + v^2,$$

prove that JJ' = 1.

(*b*) Find the percentage error in computing the parallel resistance r of three resistances r_1, r_2, r_3 from the formula :   [4]

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

if r_1, r_2, r_3 are in error by 2% each.

(*c*) Find the stationary points of :

$$T(x, y, z) = 8x^2 + 4yz - 16z + 600$$

if the condition $4x^2 + y^2 + 4z^2 = 16$ is satisfied.   [5]