Total No. of Questions : 9]

SEAT No. :

P9095

[Total No. of Pages : 4

[6179]-220

S.E. (Electrical)

ENGINEERING MATHEMATICS-III

(2019 Pattern) (Semester-III) (207006)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Question 1 is compulsory.*
2) *Attempt Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q9.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Black Figures to the right indicate full marks.*
5) *Use of logarithmic tables slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.*
6) *Assume suitable data if necessary.*

Q1) Choose the correct option.

a) If $U(k)=\begin{cases}0 & k<0\\ 1 & k\geq 0\end{cases}$ the $z\{U(k)\}$ is given by **[2]**

i) $\frac{-z}{z-1}, |z|>|$

ii) $\frac{1}{z-1}, |z|>|$

iii) $\frac{z}{z-1}, |z|>|$

iv) $\frac{2}{z-1}, |z|>|$

b) If $f(x)$ defined in the interval $-\infty<x<\infty$ is an even function, then fourier cosine transform of $f(x)$ is, **[1]**

i) $F_c(\lambda)=\int_0^\infty f(u)\cos\lambda u\, du$

ii) $F_c(\lambda)=\int_{-\infty}^{\infty} f(u)\cos\lambda u\, du$

iii) $F_c(\lambda)=\frac{\pi}{2}\int_0^\infty f(u)\cos\lambda u\, du$

iv) $F_c(\lambda)=\int_0^\infty f(u)\sin\lambda u\, du$

c) Standard deviation of four numbers 9,11,13,15, is, **[2]**

i) 2

ii) 4

iii) $\sqrt{6}$

iv) $\sqrt{5}$

d) Mean of Binomial Distribution with parameters n &p is,____ **[1]**

i) nq

ii) n^2p

iii) npq

iv) np

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e) For constant vector $\overline{a}$, $\nabla \times (\overline{a} \times \overline{r}) =$ ______ **[2]**

i) $3\overline{a}$ ii) $\overline{a}$

iii) 0 iv) $2\overline{a}$

f) Residue of $\dfrac{z+1}{z^2+1}$ at the pole $z = i$ is, **[2]**

i) $\dfrac{i-1}{2i}$ ii) $\dfrac{1-i}{2}$

iii) $\dfrac{1+i}{2i}$ iv) $\dfrac{1-i}{2i}$

Q2) a) Attempt any one **[4]**

i) Find z transform of $f(k) = \left(\dfrac{1}{4}\right)^{|k|}$, for all k

ii) Find inverse z-transform of $f(z) = \dfrac{1}{(z-2)(z-3)}, |z| > 3$

b) Obtain $f(k)$; given that **[6]**

$$f(k+1) + \frac{1}{2} f(k) = \left(\frac{1}{2}\right)^k, k \geq 0, f(0) = 0$$

c) Find the fourier cosine transform of the function. **[5]**

$$f(x) = \begin{cases} \cos x & 0 \leq x \leq a \\ 0 & x > a \end{cases}$$

OR

Q3) a) Attempt any one. **[5]**

i) Find z transform of $f(k) = 4^k \sin(2k+3), k \geq 0$

ii) Find inverse z transform of $f(z) = \dfrac{z(z+1)}{z^2 - 2z + 1}, |z| > 1$

b) Find fourier cosine transform of **[5]**

$$f(x) = \begin{cases} x^2 & 0 < x < a \\ 0 & x > a \end{cases}$$

c) Solve the following integral equation **[5]**

$$\int_0^\infty f(x) \sin \lambda x \, dx = \begin{cases} 1 & 0 \leq \lambda < 1 \\ 2 & 1 \leq \lambda < 2 \\ 0 & \lambda \geq 2 \end{cases}$$

Q4) a) The first four moments of a distribution about the value 2 are –1.1, 89, –110 and 23,300. Obtain the first four central moments, β_1 and β_2. **[5]**

b) Obtain the correlation coefficient for the following data. **[5]**

x	3	4	6	8	10
y	10	7	8	8	6

c) A fair coin is tossed 5 times. What is the probability of getting at least two tails? **[5]**

OR

Q5) a) Obtain the line of regression of y on x for the following data. **[5]**

x	3	4	6	8	10
y	2	4	5	7	8

b) The number of accidents per week on a highway follows a poisson distribution with mean 0.5. Find the probability that during a week there will be at the most one accident. **[5]**

c) The lifetime of an article has a normal distribution with mean 400 hours and standard deviation 50 hours. Assuming normal distribution, find the expected number of articles out of 2,000 whose life time lies between 335 hours to 465 hours. **[5]**

[Given : $z = 1.3$, $A = 0.4032$]

Q6) a) Find the directional derivative of $\phi = 3 \log (x + y + z)$ at (1, 1, 1) in the direction of tangent to the curve $x = b$ sint, $y = b$ cost, $z = bt$, at $t = 0$. **[5]**

b) If the vector field. $\overline{F} = (x+2y+az)\overline{i} + (bx-3y-z)\overline{j} + (4x+cy+2z)\overline{k}$ is irrotational find a, b, c and determine ϕ such that $\overline{F} = \nabla\phi$. **[5]**

c) If $\overline{F} = (2xy+3z^2)\overline{i} + (x^2+4yz)\overline{j} + (2y^2+6xz)\overline{k}$, evaluate $\int_c \overline{F}\cdot d\overline{r}$ where C is the curve $x = t$, $y = t^2$, $z = t^3$. joining (0, 0, 0) and (1, 1, 1). **[5]**

OR

Q7) a) Find the directional derivative of $\phi = xy^2 + yz^3$ at (1, –1, 1) towards the point (2, 1 ,–1). **[5]**

b) Prove (any one) **[5]**

i) $\nabla^4 e^r = e^r + \frac{4}{r} e^r$

ii) $\nabla\left(\bar{a} \cdot \nabla \frac{1}{r}\right) = \frac{3(\bar{a} \cdot \bar{r})\bar{r}}{r^5} - \frac{\bar{a}}{r^3}$.

c) Using Green's theorem evaluate $\int_c \overline{F} \cdot d\bar{r}$ where $\overline{F} = \left(3y\bar{i} + 2x\bar{j}\right)$ and c is the boundary of region bounded by $y = 0$, $y = \sin x$, $x = 0$, $x = \pi$. **[5]**

Q8) a) If $u = x^3 + 3y^2x$, find it's harmonic conjugate v. Also find $f(z) = u + iv$ in terms of z. **[5]**

b) Evaluate $\oint_C \frac{2z^2 + z + 5}{\left(z - \frac{1}{2}\right)^2} dz$,where 'C' is the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. **[5]**

c) Find the bilinear transformation, which sends the points 1, i, –1 from z-plane into the points i, 0, $-i$, of the w–plane. **[5]**

OR

Q9) a) Determine k such that the function $f(z) = e^x \cos y + i e^x \sin ky$ is analytic. **[5]**

b) Applying residue theorem evaluate $\oint_C \frac{z + 2}{z^2 + 1} dz$ where 'C' is the curve $|z - i| = \frac{1}{2}$. **[5]**

c) Find the map of the straight line $y = x$ under the transformation $w = \frac{z - 1}{z + 1}$. **[5]**

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