Total No. of Questions : 9]

SEAT No. :

P-1501

[Total No. of Pages : 5

[6002]-129

S.E. (Electrical)

Engineering Mathematics - III

(2019 Pattern) (Semester - III) (207006)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Question No. 1 is compulsory.*
2) *Attempt Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.*
3) *Assume suitable data if necessary.*
4) *Neat diagrams must be drawn wherever necessary.*
5) *Figures to the right side indicate full marks.*
6) *Use of electronic pocket calculator is allowed.*

Q1) a) The standard deviation of binomial probability distribution is, **[1]**

i) $\sqrt{pq}$ ii) $\sqrt{np}$

iii) np iv) $\sqrt{npq}$

b) The poles of $f(z)=\dfrac{e^z}{(z-1)(z-\frac{3}{2})}$ are **[1]**

i) $z = 1,2$ ii) $z = 3, -1$

iii) $z = 3/2, -1$ iv) $z = 1, 3/2$

c) In a poisson probability distribution if $n = 100$, $p = 0.01$ $p(r = 0)$ is given by, **[2]**

i) $2/e$ ii) $4/e$

iii) $1/e$ iv) $3/e$

d) If $\phi = 2x^2 - 3y^2 + 4z^2$, then curl (grad ϕ) is, **[2]**

i) 3 ii) $4x\hat{i} - 6y\hat{j} + 8z\hat{k}$

iii) 0 iv) $4x - 6y + 2z$

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e) Residue of $f(z)=\dfrac{z+1}{z^2+1}$ at a pole $z = i$ is, **[2]**

i) $\dfrac{i+1}{2i}$ ii) $\dfrac{1-i}{2}$

iii) $\dfrac{i-1}{2i}$ iv) $\dfrac{1-i}{2i}$

f) z-transform of $f(k) = \dfrac{2^k}{k!}$ $k \geq 0$ is given by **[2]**

i) $e^{z/2}$ ii) e^{2z}

iii) e^{z} iv) $e^{2/z}$

Q2) a) Find the fourier sine transform of $\dfrac{e^{-ax}}{x}$. **[5]**

b) Attempt any one : **[5]**

i) Find z-transform of $f(k) = 3(2^k) - 4(3^k)$ $k \geq 0$

ii) Find the inverse z-transform of $f(z)=\dfrac{z}{(z-2)(z-3)}$; $|z|>3$

c) Solve $f(k+2) - 5f(k+1) + 6f(k) = 36$, $f(0) = f(1) = 0$. **[5]**

OR

Q3) a) Attempt any one : **[5]**

i) Find z-transform of $f(k)=\dfrac{2^k}{k}, k\geq 1$

ii) Find inverse z-transform of

$$f(z) = \frac{z^2}{(z-1)\left(z-\frac{1}{2}\right)} ; \frac{1}{2}<|z|<1$$

b) Solve the integral equation : **[5]**

$$\int_0^\infty f(x)\sin\lambda x\,dx = \begin{cases} 1 & 0\le\lambda<1 \\ 2 & 1\le\lambda<2 \\ 0 & \lambda\ge 2 \end{cases}$$

c) Find the fourier transform of **[5]**

$$f(x)=\begin{cases} a-|x| & |x|\le a \\ 0 & |x|>a \end{cases}$$

Hence, find the value of $\int_0^\infty \frac{\sin^2 x}{x^2}dx$

Q4) a) First four moments about the value 4 are 1, 4, 10 and 45. Find first four moments about mean, coefficient of skewness and kurtosis. **[5]**

b) Find correlation coefficient for following distribution, **[5]**

x	5	9	15	19
y	7	9	14	21

c) The mean and variance of Binomial distribution are 4 and 2 respectively, Find P($r \le 2$). **[5]**

OR

Q5) a) Given the information, $n = 5$, $\Sigma x = 30$, $\Sigma y = 40$, $\Sigma x^2 = 220$, $\Sigma y^2 = 340$, $\Sigma xy = 214$. Find regression line of y on x and estimate y for $x = 10$. **[5]**

b) The mean number of defectives in a sample of 20 is 2 Out of 2000 such samples, how many would be expected to contain, i) No defective sample ii) At most 3 defective, samples. **[5]**

c) A fair coin is tossed 64 times. Using normal distribution with mean 32, standard deviation 4 find the probability of getting i) number of heads between 28 to 40 and ii) number of heads less than 28. **[5]**

[Given : A(1) = 0.3413, A(2) = 0.4772]

Q6) a) Find directional derivative of $\phi = xy^2 + yz^3$ at (1, –1, 1) along the direction normal to the surface $x^2 + y^2 + z^2 = 9$ at (1, 2, 2) **[5]**

b) Show that the vector Field. **[5]**

$\overline{F}=(y\sin z - \sin x)\hat{i} + (x\sin z+2yz)\hat{j} + (xy\cos z+y^2)\hat{k}$ is irrotational, and find scalar potential ϕ such that $\overline{F}=\nabla\phi$.

c) Find the work done in moving a particle once round the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1, z = 0$ under the force field. **[5]**

$\overline{F}=(2x-y+z)\hat{i}+(x+y-z^2)\hat{j}+(3x-2y+4z)\hat{k}$

OR

Q7) a) Find directional derivative of $\phi = 4y^2z - 2xz^3$ at (1, 2, –1) along the line $x - 1 = 2(y + 1) = z - 2$ **[5]**

b) Show that (any one) : **[5]**

i) $\nabla.\left[r\nabla\left(\frac{1}{r^3}\right)\right]=3/r^4$

ii) $\nabla\times\left[\frac{\overline{a}\times\overline{r}}{r^3}\right]=-\frac{\overline{a}}{r^3}+\frac{3(\overline{a}.\overline{r})}{r^5}\overline{r}$

c) Using Green's theorem for $\overline{F}=xy\hat{i}+y^2\hat{j}$ over region R enclosed by parabola $y = x^2$ and line $y = x$ in the first quadrant evaluate $\int_C (xydx+y^2dy)$. **[5]**

Q8) a) If $V = 3x^2y - y^3$, find its harmonic conjugate u. Also find $f(z) = u + iv$ interms of z. **[5]**

b) Use Cauchy's integral formula to evaluate $\oint_C \frac{4z^2+z}{(z-1)(z+1)}dz$ where C is the circle $|z-1|=\frac{1}{2}$ **[5]**

c) Find a bilinear transformation which maps the points $-i$, 0, $2 + i$ of the z-plane onto the points 0, $-2i$, 4 of the w-plane. **[5]**

OR

Q9) a) If $f(z) = u + iv$ is analytic, find $f(z)$ if $u - v = (x - y)(x^2 + 4xy + y^2)$. **[5]**

b) Evaluate $\oint_C \frac{e^z}{(z+1)}\frac{dz}{(z+2)}$, where C is the circle $|z+1| = \frac{1}{2}$ using Cauchy's Integral formula. **[5]**

c) Find the map of the straight line $y = x$ under the transformation $w = \frac{z-1}{z+1}$ **[5]**