Total No. of Questions—8] [Total No. of Printed Pages—4

Seat No.	

[5352]-541

S.E. (Electrical Engg./Instru. & Control) EXAMINATION, 2018

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

N.B. :— (*i*) Figures to the right indicate full marks.

(*ii*) Use of electronic pocket calculator is allowed.

(*iii*) Net diagrams must be drawn wherever necessary.

(*iv*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(1) $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = \sin 2x$

(2) $(D^2 + 9)y = \frac{1}{1 + \sin 3x}$ by variation of parameters method

(3) $(4x + 1)^2 \frac{d^2y}{dx^2} + 2(4x + 1)\frac{dy}{dx} + y = 2x + 1$

(*b*) Solve by Laplace Transform method [4]

$$\frac{d^2y}{dt^2} + 9y(t) = 18t$$

with $y(0) = 0$, $y(\pi/2) = 0$.

Or

2. (*a*) An inductor of 0.5 henry is connected in series with resistor of 6 ohms. A capacitor of 0.02 farad and generator having alternative voltage given by 24 sin $10t$ $(t > 0)$ with a switch K. Forming a differential equation find the current and charge at any time t if charge is zero when switch is closed at $t = 0$. [4]

(*b*) Solve any *one* : [4]

(1) $L\left[t\int_0^t e^{-4t} \sin 3t \, dt\right]$

(2) $L^{-1}\left[\frac{2s+1}{(s^2+s+1)^2}\right]$

(*c*) Find Laplace transform of $(1 + 2t - 3t^2 + 4t^3) \cup (t - 2)$. [4]

3. (*a*) Find Fourier sine transform of $f(x) = \begin{cases} x & 0 \le x \le 1 \\ 2 - x & 1 \le x \le 2 \\ 0 & x > 2 \end{cases}$ [4]

(*b*) Attempt any *one* : [4]

(*i*) Find *z*-transform of $f(k) = (k + 1)(k + 2)2^k$, $k \ge 0$.

(*ii*) Show that

$$z^{-1}\left\{\frac{1}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}\right\} = \{x_k\} \text{ for } |z| > \frac{1}{2}$$

where $x_k = 6\left[\left(\frac{1}{2}\right)^{k-1} - \left(\frac{1}{3}\right)^{k-1}\right]$, $k \ge 1$.

(*c*) Find directional derivative of $\phi = xy^2 + yz^3$ at $(2, -1, 1)$ along the line $2(x - 2) = (y + 1) = (z - 1)$. [4]

Or

4. (*a*) Prove any *one* : [4]

(*i*) $\bar{a}.\nabla\left[\bar{b}.\nabla\left(\frac{1}{r}\right)\right] = \frac{3(\bar{a}.\bar{r})(\bar{b}.\bar{r})}{r^5} - \frac{(\bar{a}.\bar{b})}{r^3}$

(*ii*) $\nabla 4 e^r = e^r + \frac{4}{r} e^r$

(*b*) Show that $\bar{F} = (6xy + z^3)\bar{i} + (3x^2 - z)\bar{j} + (3xz^2 - y)\bar{k}$ is irrotational and find ϕ such that $\bar{F} = \nabla\phi$. [4]

(*c*) Solve $y_k - \frac{5}{6} y_{k-1} + \frac{1}{6} y_{k-2} = \left(\frac{1}{2}\right)^k \quad k \geq 0$. [4]

5. Attempt any *two* :

(*a*) Evaluate $\int_c \bar{F}.d\bar{r}$ for $\bar{F} = (2x + y)\bar{i} + (3y - x)\bar{j}$ and c is the straight line joining (0, 0) and (3, 2). [6]

(*b*) Apply Stokes' theorem to evaluate

$$\int_c 4y\,dx + 2z\,dy + 6y\,dz$$

where c is the curve of intersection of $x^2 + y^2 + z^2 = 6z$ and $z = x + 3$. [7]

(*c*) Evaluate $\iint_s \bar{r}.\hat{n}\,d\bar{s}$ over the surface of a sphere of radius 1 with centre at the origin. [6]

Or

6. Attempt any *two* :

(*a*) Using Green's theorem evaluate $\int_c \bar{F}.d\bar{r}$ where

$$\bar{F} = (2x - \cos y)\bar{i} + x(4 + \sin y)\bar{j}$$

where c is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \; z = 0$. [6]

(*b*) Evaluate $\iint_s (\nabla \times \overline{F}).d\overline{s}$ for $\overline{F} = y\overline{i} + z\overline{j} + x\overline{k}$ where s is the surface of paraboloid $z = 1 - x^2 - y^2$ above the XOY plane. [7]

(*c*) Use Gauss divergence theorem to evaluate $\iint_s \overline{F}.d\overline{s}$ over the cylindrical region bounded by $x^2 + y^2 = 4$, $z = 0$, $z = \alpha$, where $\overline{F} = x\overline{i} + y\overline{j} + z^2\overline{k}$. [6]

7. (*a*) If V = sinh x cos y find u such that $u + iv$ is analytic function. [4]

(*b*) Evaluate $\oint_c \frac{1+z}{z(z-2)}\, dz$ where c is the circle $|z| = 1$. [4]

(*c*) Find the bilinear transformation which maps points 1, i, –1 of z-plane onto i, o, $-i$ of w-plane. [5]

Or

8. (*a*) Find 'a' such that the function $f(z) = r^2 \cos 2\theta + ir^2 \sin (a\theta)$ is an analytic function. [4]

(*b*) Evaluate $\oint_c \frac{15z+9}{z(z+3)}\, dz$ where c is the circle $|z - 1| = 3$. [4]

(*c*) Show that under the transformation $w = \frac{i-z}{i+z}$, x-axis in z-plane is mapped onto the circle $|w| = 1$. [5]