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[5057]-2001

S.E. (Civil Engineering) (First Semester) EXAMINATION, 2016

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours

Maximum Marks : 50

N.B. :— (i) Attempt Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4,
Q. No. 5 or Q. No. 6 and Q. No. 7 or Q. No. 8.

(ii) Draw neat diagrams wherever needed.

(iii) Figures to the right indicate full marks.

(iv) Assume suitable data if necessary.

(v) Use of non-programmable pocket calculator is allowed.

1. (a) Solve any *two* of the following : [8]

(i) $(D - 1)^3 y = e^x + 2^x - \frac{3}{2}$

(ii) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{1}{1 + e^x}$

(iii) $(1 + x)^2 \frac{d^2y}{dx^2} + (1 + x) \frac{dy}{dx} + y = 2 \sin \log (1 + x)$

P.T.O.

- (b) Apply Gauss elimination method to solve : [4]

$$x + 4y - z = 5$$

$$x - y - 6z = -12$$

$$3x - y - z = 4.$$

Or

2. (a) The deflection of a strut with one end built in ($x = 0$) and other supported and subjected to end thrust P satisfies the equation $\frac{d^2y}{dx^2} + a^2y = \frac{a^2R}{P} (l-x)$. Given that $\frac{dy}{dx} = y = 0$ when $x = 0$ and $y = 0$, when $x = l$. Prove that :

$$y = \frac{R}{P} \left[\frac{\sin ax}{a} - l \cos ax + l - x \right]$$

where $al = \tan al$. [4]

- (b) Apply Runge-Kutta method of fourth order to solve

$$10 \frac{dy}{dx} = x^2 + y^2; y(0) = 1$$

for $x = 0.1$ taking $h = 0.1$. [4]

- (c) Solve the following system by using triangularisation method :

$$x_1 + x_2 - x_3 = 2$$

$$2x_1 + 3x_2 + 5x_3 = -3$$

$$3x_1 + 2x_2 - 3x_3 = 6$$
 [4]

3. (a) Calculate the coefficient of correlation between the marks obtained by 8 students in Mathematics and Statistics : [4]

Mathematics Marks	Statistics Marks
25	08
30	10
32	15
35	17
37	20
40	23
42	24
45	25

- (b) If mean and variance of a binomial distribution are 4 and 2 respectively, find probability of : [4]
- (i) exactly 2 successes and
- (ii) less than 2 successes.
- (c) Find the directional derivative of the function $\phi = x^2 - y^2 + 2z^2$ at the point, P (1, 2, 3) in the direction of line PQ, where Q is the point (5, 0, 4). [4]

Or

4. (a) If the first four moments of a distribution about the value 5, are equal to -4 , 22 , -117 and 560 , determine the central moments and β_1 and β_2 . [4]

(b) Attempt any one : [4]

(i) $\nabla^2(r^n \log r) = [n(n+1) \log r + 2n+1]r^{n-2}$

(ii) Evaluate $\nabla \cdot \left[r \nabla \left(\frac{1}{r^3} \right) \right]$.

(c) Show that the vector field given by [4]

$$\bar{A} = [x^2 + xy^2]\bar{i} + (y^2 + x^2y)\bar{j}$$

is irrotational and find the scalar potential such that $\bar{A} = \nabla\phi$.

5. Attempt any two :

(a) Evaluate $\int_C \bar{F} \cdot d\bar{r}$, where $\bar{F} = (2x + y^2)\bar{i} + (3y - 4x)\bar{j}$ along the parabolic arc $y^2 = x$ joining $(0, 0)$ to $(1, 1)$. [6]

(b) Evaluate $\iint_S \bar{F} \cdot d\bar{S}$, where $\bar{F} = yz\bar{i} + zx\bar{j} + xy\bar{k}$ and S is the part of the surface of the sphere $x^2 + y^2 + z^2 = 1$ which lies in the first octant. [6]

(c) Evaluate $\iint_S \nabla \times \bar{F} \cdot \hat{n} dS$ for the surface of the paraboloid $z = 4 - x^2 - y^2$, $z \geq 0$ and $\bar{F} = y^2\bar{i} + z\bar{j} + xy\bar{k}$. [7]

Or

6. Attempt any two :

(a) Find the work done in moving a particle once round the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$$

under the field of force given by

$$\bar{F} = (2x - y + z)\bar{i} + (x + y - z^2)\bar{j} + (2x - 3y + 4z)\bar{k}. \quad [6]$$

(b) Show that

$$\int_C [\bar{u} \times (\bar{r} \times \bar{v})] \cdot d\bar{r} = -(\bar{u} \times \bar{v}) \cdot \iint_S d\bar{S},$$

where S is the open surface bounded by closed curve C and $\bar{u}$ and $\bar{v}$ are constant vectors. [6]

(c) Evaluate :

$$\iint_S (y^2 z^2 \bar{i} + z^2 x^2 \bar{j} + x^2 y^2 \bar{k}) \cdot d\bar{S},$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = 4$ in the positive octant. [7]

7. Solve any *two* of the following :

(a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y(x, 0) = y_0 \sin^3 \left(\frac{\pi x}{l} \right)$. If it is released from this position, find the displacement y at any distance x from one end and at any time t . [7]

(b) Solve the one-dimensional heat flow equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ subject to the following conditions :

(i) $u(0, t) = 0$

(ii) $u(l, t) = 0$ for all t ,

(iii) $u(x, 0) = x, 0 \leq x \leq l/2$

$$= l - x, \frac{l}{2} \leq x \leq l.$$

(iv) $u(x, t)$ is bounded. [6]

- (c) A thin sheet of metal is bounded by x -axis and lines $x = 0$ and $x = l$ and stretched to infinity in y direction has its upper and lower faces perfectly insulated and its vertical edges and edge at infinity are maintained at constant temperature 0°C , while the temperature on the short edge $y = 0$ is maintained at 100°C . Find the steady state temperature $u(x, y)$. [6]

Or

8. Solve any *two* of the following :

- (a) A string is stretched tightly between $x = 0$ and $x = l$ and both ends are given displacement $y = a \sin pt$ perpendicular to the string. If the string satisfies the differential equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$, prove that the oscillations of the string are given by :

$$y = a \sec \frac{pl}{2c} \cos \left(\frac{px}{c} - \frac{pt}{2c} \right) \sin pt. \quad [7]$$

- (b) Solve the following one-dimensional heat flow equation :

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \quad \text{subject to the following conditions :}$$

(i) $u(0, t) = 0$

(ii) $u(l, t) = 0$ for all t ,

(iii) $u(x, 0) = x$, $0 < x < l$, and $u(x, t)$ is bounded. [6]

- (c) A thin metal plate bounded by the x -axis and the lines $x = 0$ and $x = 1$ and stretching to infinity in y -direction has its upper and lower faces perfectly insulated and its vertical edges and edge at infinity are maintained at the constant temperature 0°C , while over the base temperature of 50°C is maintained. Find the steady state temperature $u(x, y)$. [6]